MD02 (cont)

Q	Solution					Marks	Total	Comments
2(a)	2	4	0	5	5			
	4	2	0	4	3	M1		rows reduced (allow one slip)
	5	0	1	9	2			
	1	1	0	7	4			
	0	2	0	3	5			
	2	4	0	2	3			
	4	2	0	1	1	m1		columns reduced next
	5	0	1	6	0			Correct table
	1	1	0	4	2	A1	3	$k=6$ stated or correct in table
	0	2	0	0	3			
(b)	3 lines needed to cover zeros shown					B1		middle column, middle and bottom rows
	Reduce each uncovered element by 1 and increase double covered by 1					M1		Condone one slip
	1	3	0	1	2			
	3	1	0	0	0			
	5	0	2	6(k)	0			
	0	0	0	3	1	A1	3	FT "their k". Condone k instead
	0	2	1	0	3			of 6
(c)	A3					M1		Or correct "rings" round elements for one complete solution
	(A3)	B4	C5	D2	E1	A1		first correct matching – must be stated
	(A3)	B5	C2	D1	E4	A1	3	second correct matching and no others
(d)	Minimum total penalty points = 22					B1	1	
					Total		10	

JAN 11

MD02 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)				
	4 8 12 2 6 0 5 12 4 8 11 10 8 3 8 2 9 3 5 1 n n n n n	B1	1	
(ii)	No of rows = no of columns Hungarian algorithm minimises $20 - x$ gives measure of criteria not met which needs minimising	E1 E1 E1	3	square matrix by adding extra row (total score) points lost (in each entry)
(b)(i)	2 6 10 0 4 0 5 12 4 8 8 7 5 0 5 1 8 2 4 0 0 0 0 0 0	M1 A1✓	2	reducing rows column reduction leaves matrix unchanged ($p = 4$, $q = 5$) (ft one slip)
(ii)	Zeros covered with 4 lines shown	B1		row 5 and columns 1, 4 and 5
	2 4 8 0 4 0 3 10 4 8 8 5 3 0 5 1 6 0 4 0 2 0 0 2 2	M1 A1		subtract 2 from all uncovered and add 2 to double covered (condone one slip) (follow through their p and q)
		M1		augment (at least) one more time (condone one slip)

	2 1 5 ⟨0⟩ 1
	⟨0⟩ 0 7 4 5
	8 2 ⟨0⟩ 0 2
	4 6 0 7 ⟨0⟩
	5 ⟨0⟩ 0 5 2
		A1		may put line through second row and not
				first column
				    0 2 6 0 2 0 1 5 0 1
				    0 3 10 6 8 0 2 9 6 7
				M1 6 3 1 0 3 → 6 2 0 0 2 A1
				    1 6 0 6 0 2 6 0 7 0
				    2 0 0 4 2 3 0 0 5 2
				
				any correct final matrix requiring zeros to
				be covered by 5 lines

	1D, 2A, 3C, 4E is matching	B1	6	(field B unused)
(iii)	$(18 + 20 + 12 + 19 =)\ 69$	B1	1	
	Total		13	

JUNE 2011

Q	Solution					Marks	Total	Comments
2(a)	3	1	0	4	1	M1		reducing columns first
	1	4	1	2	4			
	1	0	3	1	2			
	2	3	2	0	0			
	0	5	1	2	1			
	3	1	0	4	1			then rows
	0	k	0	1	3			$k = 3$ stated or value 3 in table
	1	0	3	1	2			
	2	3	2	0	0			
	0	5	1	2	1	A1cso	2	**AG**
(b)(i)	Lines through columns 1, 2, 3 and row 4					B1	1	
(ii)	3	1	0	3	0			
	0	3	0	0	2	M1		subtract 1 from all uncovered and add 1 to all double covered (condone one slip)
	1	0	3	0	1			
	3	4	3	0	0			
	0	5	1	1	0	A1	2	all correct ISW
	This now requires 5 lines to cover zeros							
(c)	A2	B3	C1	D4	E5	B1		one of these correct
	A5	B3	C1	D2	E4	B1		second way correct
	A5	B3	C2	D4	E1	B1	3	third way correct and no others
(d)	Minimum total = 68 (mins)					B1	1	
(e)	Replace each element x by $N - x$					E1	1	any value of N
					Total		10	

JAN 2012

MD02 (cont)

Q	Solution	Marks	Total	Comments
2(a)	Hungarian algorithm used to find **minimum total** **Each new entry** gives measure of points **not** scored $\Rightarrow$ Hungarian algorithm now finds maximum total score	E1 E1	2	First E1– fairly generous for idea of "minimising" or "points not scored". Second E1 is strict.
(b)	Replacing x by $35 - x$ 8 6 10 0 4 2 13 18 6 6 12 6 10 2 14 13 6 6 8 4 8 8 16 14 8	B1		Must see this table
	8 6 10 0 4 0 11 16 4 4 10 4 8 0 12 9 2 2 4 0 0 0 8 6 0	M1		reducing rows; ft one slip from above & allow one further slip
	8 6 8 0 4 0 11 14 4 4 10 4 6 0 12 9 2 0 4 0 0 0 6 6 0	A1cso	3	check working is correct since most values in final table are given. ($p = 14$ $q = 9$)
(c)	Lines covering R_4 , R_5 and C_1 , C_4	B1		4 correct lines
	8 2 4 0 0 0 7 10 4 0 10 0 2 0 8 13 2 0 8 0 4 0 6 10 0	M1 A1	3	subtracting 4 from each uncovered and adding 4 to each double covered (condone 2 slips) all correct
(d)(i)	B1 and D3 A4 B1 C2 D3 E5 A5 B1 C4 D3 E2	M1 A1 A1	3	or one **full** matching with rings etc one correct matching second correct and no others
(ii)	Total = 153	B1	1	
	Total		12	

JUNE 2012

MD02

Q	Solution	Marks	Total	Comments
2(a)	0 1 2 4 3 ** 3 3 ** 0 1 4 4 2 0 0 0 2 0 0 0 3 2 0 0	M1		*may have large number instead of ** throughout this question* row adjustment (condone one slip) identical numerical error in more than one term is one slip
	0 1 0 4 3 ** 3 1 ** 0 1 4 2 2 0 0 0 0 0 0 0 3 0 0 0	A1 B1	3	then columns four lines through rows 1, 4 & 5 and column 5 *ft one slip from above for next two marks*
(b)	adjustment adding 1 to double covered and − 1 to uncovered 0 1 0 4 4 ** 2 0 ** 0 0 3 1 1 0 0 0 0 0 1 0 3 0 0 1	M1 A1√ A1	3	ft 'their lines and table' provided no more than one slip in earlier table must make ⩽ 2 further errors for M1 (⩽ 1 further error in adjustment) correct
(c)	B4 and D5 allocated A1 B4 C2 D5 E3 A3 B4 C1 D5 E2	M1 A1 A1	3	(or one complete matching ringed) one correct allocation 2nd matching and no others 1A 2C 3E 4B 5D 1C 2E 3A 4B 5D
(d)	13 + 16 + 21 + 20 + 15 ⎫ or 16 + 16 + 15 + 20 + 18 ⎬ Min Total Time = 85 (min)	B1	1	
	Total		10	

JAN 2013

Q	Solution	Marks	Total	Comments
3(a)	$\begin{pmatrix} 8 & 5 & 0 & 9 & 6 \\ 5 & 6 & 5 & 9 & 7 \\ 11 & 10 & 12 & 12 & 11 \\ 9 & 5 & 8 & 12 & 9 \end{pmatrix}$	B1	1	
(b)	Add an extra row ≥ 12	B1		
	$\begin{pmatrix} 8 & 5 & 0 & 9 & 6 \\ 5 & 6 & 5 & 9 & 7 \\ 11 & 10 & 12 & 12 & 11 \\ 9 & 5 & 8 & 12 & 9 \\ 12 & 12 & 12 & 12 & 12 \end{pmatrix}\begin{matrix} (0) \\ (5) \\ (10) \\ (5) \\ (12) \end{matrix}$			
	$\begin{matrix} 8 & 5 & 0 & 9 & 6 \\ 0 & 1 & 0 & 4 & 2 \\ 1 & 0 & 2 & 2 & 1 \\ 4 & 0 & 3 & 7 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$	M1 A1		3 rows correct from row reduction All correct
	(Zeros correctly covered by 4 lines)	B1F		**Alternatives**
	Covered in 4 lines, not optimal (reduce by 1)	E1		*
		B1		
	5 lines needed, optimal	(E1)		*or earned here
	Match WC, XA, YE, ZB, (−D)	B1		
	Value = 151	B1	8	
	Total		9	

JUNE 2013

Q	Solution	Marks	Total	Comments
3(a)	Reduce columns $$\begin{pmatrix} 0 & 12 & 13 & 2 & 0 \\ 25 & 32 & 11 & 20 & 20 \\ 5 & 12 & 2 & 8 & 25 \\ 15 & 17 & 21 & 35 & 15 \\ 0 & 0 & 0 & 0 & 7 \end{pmatrix}$$ Reduce rows $$\begin{pmatrix} 0 & 12 & 13 & 2 & 0 \\ 14 & 21 & 0 & 9 & 9 \\ 3 & 10 & 0 & 6 & 23 \\ 0 & 2 & 6 & 20 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{pmatrix}$$ $k = 9$	M1 A1 B1	3	 AG
(b)	4 lines drawn on given table Subtract/add 2 $$\begin{pmatrix} 0 & 10 & 13 & 0 & 0 \\ 14 & 19 & 0 & 7 & 9 \\ 3 & 8 & 0 & 4 & 23 \\ 0 & 0 & 6 & 18 & 0 \\ 2 & 0 & 2 & 0 & 9 \end{pmatrix}$$ Subtract/add 3 $$\begin{pmatrix} 0 & 10 & 16 & 0 & 0 \\ 11 & 16 & 0 & 4 & 6 \\ 0 & 5 & 0 & 1 & 20 \\ 0 & 0 & 9 & 18 & 0 \\ 2 & 0 & 5 & 0 & 9 \end{pmatrix}$$	B1 M1 A1 m1 A1	 5	 Condone one slip Correct table with 4 lines shown Condone one slip All correct with no errors seen, including 5 lines drawn
(c)	Match *XA, WC* + *VD, YE, ZB* or *VE, YB, ZD*	M1 A1 A1	3	And no extras
(d)	525	B1	1	
	Total		12	

JUNE 2014

Q	Solution	Mark	Total	Comment
7(a)	Row minima: $(x + 4), (x + 2), (x + 5)$	M1 A1	 2	1 correct All 3 correct
(b)	<table><tr><td>4</td><td>0</td><td>2</td><td>5</td></tr><tr><td>3</td><td>1</td><td>2</td><td>0</td></tr><tr><td>3</td><td>2</td><td>0</td><td>$x-3$</td></tr><tr><td>2</td><td>$x-4$</td><td>$11-x$</td><td>0</td></tr></table> Reduce cols to give <table><tr><td>2</td><td>0</td><td>2</td><td>5</td></tr><tr><td>1</td><td>1</td><td>2</td><td>0</td></tr><tr><td>1</td><td>2</td><td>0</td><td>$x-3$</td></tr><tr><td>0</td><td>$x-4$</td><td>$11-x$</td><td>0</td></tr></table> 4 lines needed to cover 0's Match AZ, BW, CY, DX stated	M1 A1 A1 M1 A1 E1 B1	 7	Using correct/'their' row minima 3 rows correct All correct 3 rows correct All correct oe
(c)	$4x+14= 42$ $x = 7$	M1 A1	 2	Their expression $= 42$
	Total		11	